Bachelor of Computer Application (B.C.A.) Semester—II Examination

DISCRETE MATHEMATICS—II

Paper—IV

Time : Three Hours] [Maximum Marks : 50

Note :—(1) **All** questions are compulsory and carry equal marks.

(2) Draw neat and labelled diagrams wherever necessary.

EITHER

1. (A) What is Venn diagram ? Draw Venn diagram of the following problem :

In a class of 30 students, 19 are studying English, 12 are studying Hindi and 7 are both studying English and Hindi. Also find out how many students are not taking any language ? 5

(B) Define Power Set. Find out power set of the following :

(i) $\{\phi, 1\}$

(ii) $\{1, 2, 3\}$. 5

OR

(C) Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c\}$

Let $R = \{(1, a), (1, b), (2, b), (2, c), (3, b), (4, a)\}$ and

$S = \{(1, b), (2, c), (3, b), (4, b)\}$.

Compute :

(i) $\bar{R}$

(ii) $R \cap S$

(iii) $R \cup S$

(iv) R^{-1}. 5

(D) If R is an equivalence relation on set A and let $a \in A$ and $b \in A$ then prove that $a R b$ if and only if $R(a) = R(b)$. 5

EITHER

2. (A) Show by Mathematical Induction, that for all $n \geq 1$,

$$1^2 + 3^2 + 5^2 + ... + (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3}$$ 5

(B) Write the permutation :

$$P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 6 & 5 & 2 & 1 & 8 & 7 \end{pmatrix}$$

of the set $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ as a product of Transposition. 5

OR

(C) Let $A_1, A_2, ..., A_n$ be any n sets. Show by Mathematical Induction that :

$$\overline{\bigcap_{i=1}^{n} A_i} = \bigcup_{i=1}^{n} \overline{A_i}$$ 5

(D) Find sequence defined by the recurrence relation :

$C_n = {}^4C_{n-1} + {}^5C_{n-2}$

$C_1 = 2$ and $C_2 = 6$. 5

EITHER

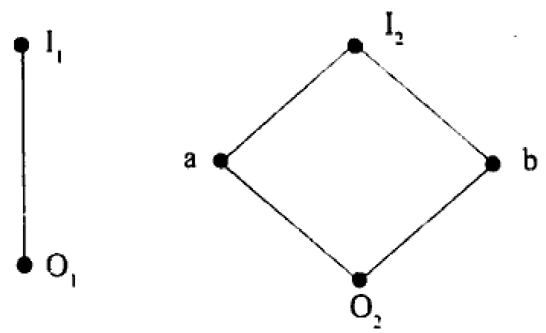
3. (A) Show that in a Boolean Algebra, for any a and b,

$((a \vee c) \wedge (b' \vee c))' = (a' \vee b) \wedge c'$. 5

(B) Let (S, *) and (T, *) be monoids with identities e and e', respectively. Let $f : S \to T$ be an isomorphism then prove $f(e) = e'$. 5

OR

(C) Let L_1 and L_2 be Lattice shown in following figures. Find $L = L_1 \times L_2$ and draw the Hasse diagram for L :



5

(D) Define the following :

(i) Group

(ii) Semigroup

(iii) Subgroup

(iv) Monoid. 5

EITHER

4. (A) Prove that, a tree with n vertices has n − 1 edges. 5

(B) Let number of edges of graph G be m, then prove that G has a Hamiltonian circuit if $m \geq \frac{1}{2}(n^2 - 3n + 6)$ where n is the number of vertices. 5

OR

(C) Prove that if a graph G has more than two vertices of odd degree, then there can be no Euler path in G. 5

(D) Explain the following :

(i) Labelled Tree

(ii) Undirected Tree. 5

5\. (A) What are the properties of Binary relation ? Explain.

(B) Explain the Pigeonhole principle.

(C) Define :

(i) Distributive Lattice

(ii) Complemented Lattice.

(D) Define Graph. Explain the representation of graph in memory. $4 \times 2\frac{1}{2} = 10$

Bachelor of Computer Application (B.C.A.) Semester—II (C.B.S.) Examination

DISCRETE MATHEMATICS–II

Paper–IV

Time : Three Hours] [Maximum Marks : 50

N.B. :— (1) **ALL** questions are compulsory and carry equal marks.

(2) Draw neat and labelled diagram wherever necessary.

EITHER

1. (A) Explain the operation of set and its types with examples. 5

(B) Prove that $A - (A \cap B) = A - B$ for the sets A and B. 5

OR

(C) Let $A = \{1, 2, 3, 4\}$ and R is relation statement $R = \{(x, y)/\ x > y\}$ find M_R and draw diagram. 5

(D) If $A = \{1, 2, 3, 4\}$

$R = \{(1, 1), (1, 2), (2, 3), (2, 4), (3, 4), (4, 1), (4, 2)\}$.

Find R^R and R^∞. 5

EITHER

2. (A) State and prove the pigeonhole principle with example. 5

(B) Prove by Mathematical induction $p(n) = 5 + 10 + 15 + \ldots\ldots\ldots + 5n = \frac{5n(n+1)}{2}$. 5

OR

(C) Let $A_1, A_2, \ldots\ldots, A_n$ be any n sets. Show by Mathematical induction that :

$$\overline{\bigcap_{i=1}^{n} A_i} = \bigcup_{i=1}^{n} \overline{A_i}$$ 5

(D) Find sequence defined by the recurrence relation :

$c_n = {}^4c_{n-1} + {}^5c_{n-2}$

$c_1 = 2$ & $c_2 = 6$ 5

EITHER

3. (A) If F is homorphism from commutative semigroup (5, *) onto semigroup (T, *) then (T, $*^{-1}$) is also commutative. 5

(B) Show that in a Boolean Algebra, for any a and b,

$((a \vee c) \wedge (b' \vee c))' = (a' \vee b) \wedge c'$. 5

OR

(C) Let $f : G \rightarrow G^{-1}$ be a group of homomorphism prove that f is one to one if and only if $\ker(f) = \{e\}$. 5

(D) Let (S, $*$) and (T, $*$) be monoids with identities e and e', respectively. Let $f : S \rightarrow T$ be an isomorphism then prove f(e) = e'. 5

EITHER

4. (A) Define :

(1) Graph

(2) Subgraph

(3) Quotient graph

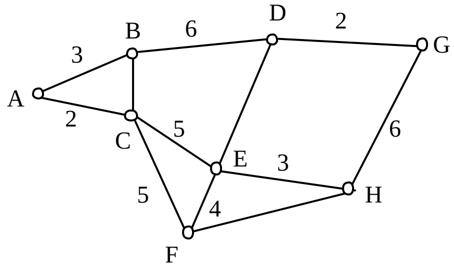
(4) Connected graph

(5) Degree of the vertex. 5

(B) Prove that if G is connected and has exactly two vertices of odd degree, there is an Euler Path in G. 5

OR

(C) Find the Hamiltonian circuit for the graph given below :



5

(D) Let L be a lattice, prove that $a \vee (b \vee c) = (a \vee b) \vee c$. 5

5. Attempt **ALL** :

(A) Explain equivalence classes with example. 2½

(B) Determine, whether the permutation is even or odd :

A = {1, 2, 3, 4, 5, 6, 7, 8}

(1) (4, 8) (3, 5, 2, 1) (2, 4, 7, 1)

(2) (6, 4, 2, 1, 5). 2½

(C) Explain group and semigroup. 2½

(D) Explain :

(1) Trees.

(2) Labelled trees. 2½

NRT/KS/19/2222

Bachelor of Computer Application (B.C.A.) Semester—II Examination

DISCRETE MATHEMATICS—II

Paper—IV

Time : Three Hours] [Maximum Marks : 50

N.B. :— (1) All questions are compulsory and carry equal marks.
(2) Draw neat and labelled diagram wherever necessary.
(3) Assume the data wherever necessary.

EITHER

1. (A) Explain inclusion and equality of sets with suitable example. 5

(B) Let X {1, 2, 3, 4} and $R = \{\langle x, y\rangle \mid x > y\}$. Draw the Graph of R and also give its matrix. 5

OR

(C) Write properties of Binary relation in a set. Explain any two with example. 5

(D) Define following operations on set with Venn diagrams :
(i) Union
(ii) Intersection
(iii) Set difference
(iv) Symmetric difference. 5

EITHER

2. (A) Define the following functions :
(i) Onto
(ii) One-to-One
(iii) Inverse function
(iv) Composite function. 5

(B) How many different seven person committees can be formed each containing three women from an available set of twenty women and four men from an available set of thirty men ? 5

OR

(C) Prove by Mathematical Induction :

$$p(n) = 1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$$ 5

(D) Backtrack to find an explicit formula for the sequence defined by recurrence relation, $b_n = 2b_{n-1} + 1$ with initial condition $b_1 = 7$. 5

EITHER

3. (A) Define Isomorphism and Homomorphism. What procedure is used to show that two semigroups are isomorphic ? 5

(B) Write properties of Binary operations with suitable example. 5

OR

(C) Define product and quotient of group with suitable example. 5

(D) Draw the Hasse Diagram of divisibility on a set for D_{20}. 5

EITHER

4. (A) Prove that, if a Group G has a vertex of odd degree, there can be no Euler circuit in G. 5

(B) Define following terms with the help suitable example :

(i) Vertex

(ii) Degree of vertex

(iii) Edge

(iv) Isolated vertex

(v) Adjacent vertex. 5

OR

(C) Define Hamiltonian path and Hamiltonian circuit with suitable example. 5

(D) Let $A = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}$ and

Let $T = \{(v_2, v_3), (v_2, v_1), (v_4, v_5), (v_4, v_6), (v_5, v_8), (v_6, v_7), (v_4, v_2), (v_7, v_9), (v_7, v_{10})\}$

Show that T is a rooted Tree and identify the root. 5

5. (A) Define set, subset and power set. 2½

(B) State and explain pigeonhole principle. 2½

(C) Define semigroups, identity and monoids. 2½

(D) Write definition for Ordered tree and Binary tree. 2½

Bachelor of Computer Application (B.C.A.) Semester–II (C.B.S.) Examination

DISCRETE MATHEMATICS–II

Paper—IV

Time : Three Hours] [Maximum Marks : 50

Note :— (1) All questions are compulsory and carry equal marks.
(2) Draw neat and labelled diagrams wherever necessary.

EITHER

1. (A) Define :
   (i) Power set
   (ii) Discount set
   (iii) Symmetric difference
   (iv) Venn diagram
   (v) Equivalence relation. 5

   (B) Let A = {1, 2, 3} and let the relations R and S on A are :
   R = {(1,1), (1,2), (2,1), (1,3), (3,1)}
   A = {(1,1), (1,2), (2,1), (2,2), (3,3)}
   Find $\overline{R}$, R^{-1}, $R \cap S$, $R \cup S$, $\overline{S}$. 5

OR

   (C) Let R be a relation on set A = {1, 2, 3, 4, 5}
   R = {(1,2), (2,1), (3,4), (4,3), (3,5), (5,3), (4,5), (5,4), (5,5)}
   Draw the graph and the find relational matrix. 5

   (D) Let R and S be relations from A to B. If R(a) = S(a) for all a in A, then prove that R = S. 5

EITHER

2. (A) Use backtrack method to find explicit formula for the sequence defined by recurrence relation $b_n = 2b_{n-1} + 1$ with the initial condition $b_1 = 7$. 5

   (B) Suppose that a valid computer consists of seven characters, the first of which is a letter chosen from the set {A, B, C, D, E, F, G} and the remaining six characters are letters chosen from the English alphabet or digit. How many different passwords are possible ? 5

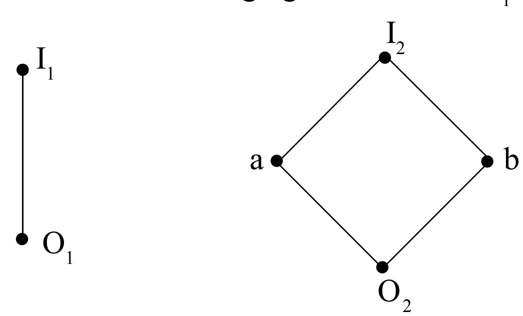
OR

   (C) Prove by mathematical induction that for all $n \geq 1$, $n! \geq 2^{n-1}$; where 1! = 1 and n! = n(n–1)! 5

   (D) Let A = {1, 2, 3, 4, 5, 6}
   Compute :
   (i) (4, 1, 3, 5) 0 (5, 6, 3)
   (ii) (5, 6, 3) 0 (4, 1, 3, 5) 5

EITHER

3. (A) Let L_1 and L_2 be lattices shown in following figures. Find $L = L_1 \times L_2$ and draw the Hasse diagram for L :



5

   (B) Let T be the set of all even integers. Show that the semigroups (z, +) and (T, +) are isomorphic. 5

OR

(C) Define the following :

(i) Group

(ii) Semigroup

(iii) Subgroup

(iv) Monoid. 5

(D) Construct the truth table for the Boolean function $f : B_3 \to B$ determined by the polynomial :

$P(x_1, x_2, x_3) = (x_1 \wedge x_2) \vee (x_1 \vee (x_2' \wedge x_3))$ 5

EITHER

4. (A) Define :

(i) Tree

(ii) Height of tree

(iii) Complete binary tree. 5

(B) Prove that, a tree with n vertices has n–1 edges. 5

OR

(C) Let A = {1, 2, 3, 4, 12}. Consider the partial order a divisibility on A. Draw the Hasse diagram of the poset $(A, \leq)$. 5

(D) Prove that :

If a graph G has more than two vertices of odd degree, then there can be no Euler path in G. 5

5. Attempt **All** :

(A) Prove that :

$A - B = A \cap \overline{B}$

for the sets A and B. 2½

(B) How many distinguishable permutations of the letters are in the word BANANA ? 2½

(C) Define groups and semigroups. 2½

(D) Define Graph and Connected graph. 2½

Bachelor of Computer Application (B.C.A.) Semester–II (C.B.S.) Examination

DISCRETE MATHEMATICS–II

Paper—IV

Time : Three Hours] [Maximum Marks : 50

Note :— (1) All questions are compulsory and carry equal marks.
(2) Draw neat and labelled diagrams wherever necessary.

EITHER

1. (A) Define :
(i) Power set
(ii) Discount set
(iii) Symmetric difference
(iv) Venn diagram
(v) Equivalence relation. 5

(B) Let A = {1, 2, 3} and let the relations R and S on A are :
R = {(1,1), (1,2), (2,1), (1,3), (3,1)}
A = {(1,1), (1,2), (2,1), (2,2), (3,3)}
Find $\overline{R}$, R^{-1}, $R \cap S$, $R \cup S$, $\overline{S}$. 5

OR

(C) Let R be a relation on set A = {1, 2, 3, 4, 5}
R = {(1,2), (2,1), (3,4), (4,3), (3,5), (5,3), (4,5), (5,4), (5,5)}
Draw the graph and the find relational matrix. 5

(D) Let R and S be relations from A to B. If R(a) = S(a) for all a in A, then prove that R = S. 5

EITHER

2. (A) Use backtrack method to find explicit formula for the sequence defined by recurrence relation $b_n = 2b_{n-1} + 1$ with the initial condition $b_1 = 7$. 5

(B) Suppose that a valid computer consists of seven characters, the first of which is a letter chosen from the set {A, B, C, D, E, F, G} and the remaining six characters are letters chosen from the English alphabet or digit. How many different passwords are possible ? 5

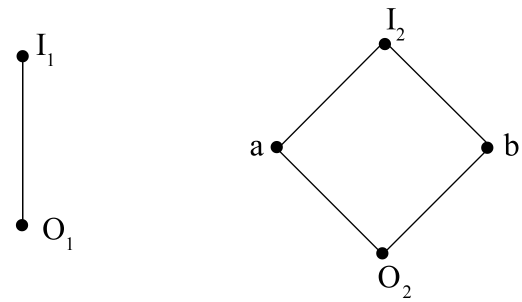
OR

(C) Prove by mathematical induction that for all $n \geq 1$, $n! \geq 2^{n-1}$; where 1! = 1 and n! = n(n–1)! 5

(D) Let A = {1, 2, 3, 4, 5, 6}
Compute :
(i) (4, 1, 3, 5) 0 (5, 6, 3)
(ii) (5, 6, 3) 0 (4, 1, 3, 5) 5

EITHER

3. (A) Let L_1 and L_2 be lattices shown in following figures. Find $L = L_1 \times L_2$ and draw the Hasse diagram for L :



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(B) Let T be the set of all even integers. Show that the semigroups (z, +) and (T, +) are isomorphic. 5

OR

(C) Define the following :

(i) Group

(ii) Semigroup

(iii) Subgroup

(iv) Monoid. 5

(D) Construct the truth table for the Boolean function $f : B_3 \rightarrow B$ determined by the polynomial :

$P(x_1, x_2, x_3) = (x_1 \wedge x_2) \vee (x_1 \vee (x_2' \wedge x_3))$ 5

EITHER

4\. (A) Define :

(i) Tree

(ii) Height of tree

(iii) Complete binary tree. 5

(B) Prove that, a tree with n vertices has n–1 edges. 5

OR

(C) Let A = {1, 2, 3, 4, 12}. Consider the partial order a divisibility on A. Draw the Hasse diagram of the poset $(A, \leq)$. 5

(D) Prove that :

If a graph G has more than two vertices of odd degree, then there can be no Euler path in G. 5

5\. Attempt **All** :

(A) Prove that :

$$A - B = A \cap \overline{B}$$

for the sets A and B. 2½

(B) How many distinguishable permutations of the letters are in the word BANANA ? 2½

(C) Define groups and semigroups. 2½

(D) Define Graph and Connected graph. 2½

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Second Semester Bacholer of Computer Application (B.C.A.) Examination

Paper–IV

DISCRETE MATHEMATICS – II

Time : Three Hours] [Max. Marks : 50

N.B. : (1) All questions are compulsory and carry equal marks.
(2) Draw neat and labelled diagram whenever necessary.

EITHER

1 (A) Define power set of a set.

If A {3, 7, 2} then,

(i) What is |A|

(ii) What is |P(A)|. 5

(B) Let A = {a, b, c, d} and let R be the relation on A that has the matrix :—

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Draw the Diagraph of R. Find the Indegree and outdegree of all vertices. 5

OR

(C) Prove De Morgan's law

$\overline{A \cup B} = \bar{A} \cap \bar{B}$ 5

(D) Let A {1, 2, 3, 4, 5} and

R {(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1), (2, 3), (3, 3), (4, 4), (3, 2), (5, 5)}.

Determine, whether the relation R on set A is an equivalence relation. 5

EITHER

2. (A) How many different seven person committees can be formed, each containing three women from an available set of 20 women and four men from an available set of 30 Men ? 5

(B) Backtrack to find an explicit formula for the sequence defined by the recurrence relation $b_n = 2.\, b_{n-1} + 1$ with initial condition $b_1 = 7$. 5

OR

(C) If the characteristic equation $x^2 - r_1,\ x - r_2 = 0$ of the recurrence relation $a_n = r_1\ a_{n-1} + r_2 \cdot a_{n-2}$ has two distinct roots, s_1 and s_2, then $a_n = u.\ s_1^n$, $v.s_2^n$, where u and v depend on the initial conditions, is the explicit formula for the sequence. 5

(D) Show by mathematical induction, that for all $n \geq 1$.

$5 + 10 + 15 + \ldots + 5n = \dfrac{5n\ (n+1)}{2}$ 5

EITHER

3. (A) Define :—

(i) Semigroup

(ii) Monoid.

(iii) Subsemigroup

(iv) Group homomorphism. 5

(B) Let A {1, 2, 3, 4, 6, 8, 9, 18, 24} be ordered by divisibility.

Draw the hasse diagram of A. 5

OR

(C) Let G be the set of all nonzero real numbers and let

$$a * b = \frac{ab}{2}$$

Show that (G, $*$) is an Abelian group. 5

(D) Show that, if n is a positive integer and P^2/n, where p is a prime number, then D_n is not a Boolean algebra. 5

EITHER

4. (A) Define :—

(i) Graph

(ii) Subgraph

(iii) Quotient graph

(iv) Complete graph. 5

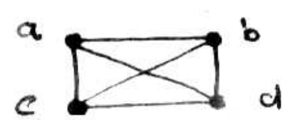
(B) Prove that

Let the number of edges of G be 'm' Then G has a Hamiltonian circuit, if $m \geq \frac{1}{2}(n^2-3n+6)$, where 'n' is the number of vertices. 5

OR

(C) Define Euler path and Euler circuit. Prove that, if a graph G has more than two vertices of odd degree, then there can be no euler path in G. 5

(D) Construct an undirected spanning tree for the following connected graph G, by removing edges in succession. Show the graph of the resulting undirected tree.



5

5. Solve any **ten** :—

(A) Define subset and disjoint set.

(B) Define antisymmetric relation

(C) Define symmetric difference of sets.

(D) State Pigeonhole principle

(E) What is bijection function?

(F) What is even and odd permutations?

(G) Let G be a group and let a, b and C be elements of G. Then state left and right cancellation property.

(H) Define Lattice.

(I) Define bounded and distributive lattice.

(J) Define Binary tree.

(K) Define Euler path and Euler circuit.

(L) Define subgraph of a graph G. 1 x 10

KNT/KW/16/5255

Bachelor of Computer Application (B.C.A) Semester-II (C.B.S.) Examination

DISCRETE MATHEMATICS—II

Paper-IV

Time : Three Hours] [Maximum Marks : 50

N.B. :— All questions are compulsory and carry equal marks.

EITHER

1. (a) A computer company wants to hire 25 programmers to handle systems programming jobs and 40 programmers for applications programming. Of those hired, ten will be expected to perform jobs of both types. How many programmers must be hired ? 5

(b) Prove that : $\overline{A \cup B} = \overline{A} \cap \overline{B}$ for the sets A and B. 5

OR

(c) Let R be an equivalence relation on set A, and let $a \in A$ and $b \in A$. Then prove that aRb if and only if $R(a) = R(b)$. 5

(d) Let A = {1, 2, 3} and let R and S be relations on A. Suppose that the matrices of R and S are

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } M_S = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Find (i) $M_{\overline{R}}$ (ii) $M_{R \cap S}$ (iii) $M_{R \cup S}$ 5

EITHER

2. (a) Let $A_1, A_2, A_3, \ldots, A_n$ be any n sets. Show by mathematical induction that

$$\left(\overline{\bigcup_{i=1}^{n} A_i}\right) = \bigcap_{i=1}^{n} \overline{A_i}$$ 5

(b) Show that ${}_{n+1}C_r = {}_nC_{r-1} + {}_nC_r$ 5

OR

(c) Use the technique of backtracking to find an explict formula for the sequence defined by the recurrence relation $b_n = 5\, b_{n-1} + 3$ with initial condition $b_1 = 2$ 5

(d) State and prove that pigeonhole principle. 5

EITHER

3\. (a) Let $(S, *)$ and $(T, *')$ be monoids with identities e and e', respectively. Let $f : S \to T$ be an isomorphism. Then prove that $f(e) = e'$ 5

(b) Let G be the set of all non zero real numbers and let $a * b = \frac{ab}{2}$.

Show that $(G, *)$ is an abelian group. 5

OR

(c) Let L be a Lattice, then for every a and b in L, prove that

$a \vee b = b$ it and only if $a \leq b$,

(d) Show that in a Boolean algebra, for any a and b, 5

$(a \wedge b) \vee (a \wedge b') = a$ 5

EITHER

4\. (a) Prove that if a graph G has more than two vertices of odd degree, then there can be no Euler path in G. 5

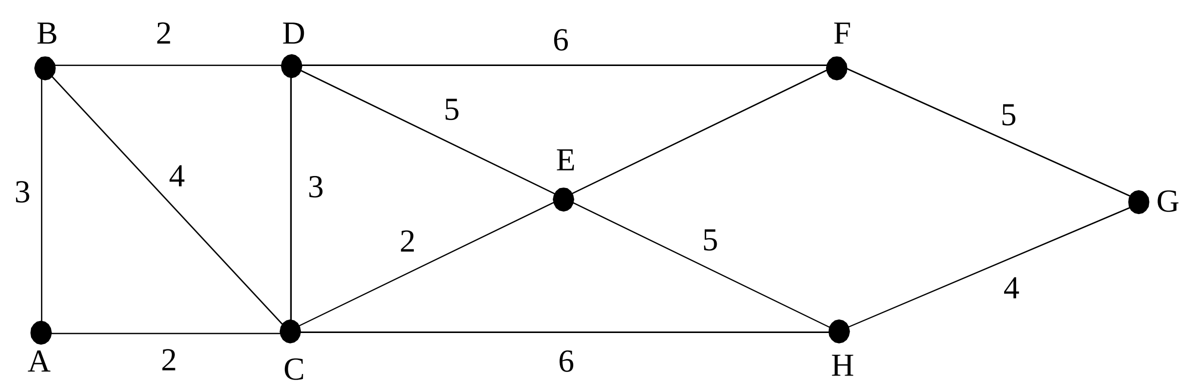
(b) Let $A = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}$

$T = \{(v_2, v_3), (v_2, v_1), (v_4, v_5), (v_4, v_6)\ (v_5, v_8), (v_6, v_7), (v_4, v_2), (v_7, v_9)\ (v_7, v_{10})\}$

Show that T is a rooted tree and identify the root. 5

OR

(c) Find the Hamiltonian circuit for the graph given below :



5

(d) Define :

(i) Graph

(ii) Subgraph

(iii) Quotient graph

(iv) Connected graph

(v) Degree of the Vertex 5

5. Attempt **all** the following :

(a) Let A = {a, b, c, d, e} and

R = {(a, a) , (a, b), (b, c), (c, e), (c, d), (d, e)}

Compute : R^2. 2½

(b) Let A = B = Z, and C be the set of even integers.

Let f : A→B and g : B→C, be defined by

f(a) = a + 1

g(b) = 2b

Find g o f. 2½

(c) Let G be a group and a be an element of G. Then show that $(a^{-1})^{-1} = a$ 2½

(d) Prove that a tree with n vertices has n–1 edges. 2½

Bachelor of Computer Application (B.C.A.) Semester—II (C.B.S.) Examination

DISCRETE MATHEMATICS—II

Paper—IV

Time : Three Hours] [Maximum Marks : 50

N.B. :— (1) **ALL** questions are compulsory and carry equal marks.

(2) Draw neat and labelled diagram wherever necessary.

EITHER

1. (a) Give the power set of following :

(i) $\{\phi, 1\}$

(ii) $\{a, b, c\}$ 5

(b) Show that for any two finite and non-empty sets A and B;

$A - (A \cap B) = A - B.$ 5

OR

(c) What do you mean by symmetric difference ? Explain with example. Also draw the Venn diagram. 5

(d) Let $A = \{a, b, c, d\}$. Let R be the relation on A, that has the matrix :

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

construct the diagraph of R and list the in-degree and out-degree of all vertices. 5

EITHER

2. (a) Prove by mathematical induction :

$1 + 2 + 3 + ... + n = n(n + 1)/2.$ 5

(b) Explain Pigeon-hole principle. 5

OR

(c) What do you mean by function ? Also explain the following functions :

(i) One to one

(ii) Onto

(iii) Inverse function. 5

(d) Find an explicit formula for the sequence defined by $C_n = 3C_{n-1} - 2C_{n-2}$ with initial conditions $C_1 = 5$ and $C_2 = 3$. 5

EITHER

3. (a) For Boolean Polynomial $P(x, y, z) = (x \wedge y) \vee (y \wedge z')$. Construct the truth table and show the Polynomial by logic diagram. 5

(b) Let $S = \{a, b, c\}$ and $A = P(S)$. Draw the Hasse diagram of the Poset with partial ordering of set inclusion. 5

OR

(c) Let L be a bounded distribution lattice. Prove that if complement of $a \in L$ exists, then it is unique. 5

(d) Let G be the set of all non-zero real numbers and let $a * b = \frac{ab}{2}$; show that $(G, *)$ is an abelian group. 5

EITHER

4. (a) Explain the following :

(i) labelled tree

(ii) undirected tree. 5

(b) Let number of edges of graph G be m, then prove that G has a Hamiltonian circuit, if $m \geq \frac{1}{2}(n^2 - 3n + 6)$, where n is the number of vertices. 5

OR

(c) Explain with the help of example :

(i) directed graph

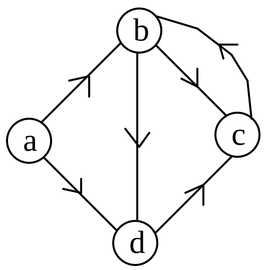
(ii) null graph

(iii) complete graph

(iv) linear graph

(v) weighted graph. 5

(d) Obtain the adjancy matrix of the diagraph given below.



5

5. Attempt **ALL** :

(a) What are the properties of binary relation ? Explain. 2½

(b) How many words can be made by using the letters of the word "BANANA", taken all at a time ? 2½

(c) For the following graph; find :

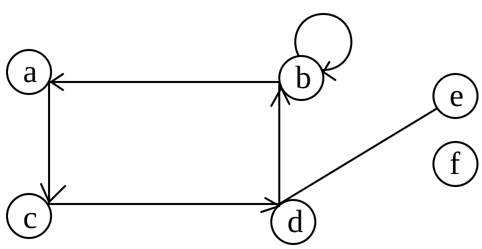
(i) vertex set

(ii) edge set

(iii) pendent vertex

(iv) loop

(v) isolated vertex.



2½

(d) Define :

(i) Distributive lattice.

(ii) Complemented lattice. 2½

TKN/KS/16/5966

Bachelor of Computer Application (B.C.A.) Part—I (Semester—II) (C.B.S.) Examination

DISCRETE MATHEMATICS—II

Paper—IV

Time—Three Hours] [Maximum Marks—50

Note :— (1) All questions are compulsory and carry equal marks.

(2) Draw neat, labelled diagrams wherever necessary.

EITHER

1. (a) Prove that $A - (A - B) \subseteq B$ where A and B are sets. 5

(b) Prove that :

Let R be an equivalence relation on A and let P be the collection of all distinct relative sets R(a) for a in A. Then P is a partition of A and R is an equivalence relation determined by P. 5

OR

(c) Prove that :

$$A - B = A \cap \overline{B}$$

for the set A and B. 5

(d) Let A = {a, b, c, d} and Let R be the relation on A that has the matrix

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Construct the diagraph of R and list indegrees and out-degree of all vertices. 5

EITHER

2. (a) Let $f : A \to B$ and $g : B \to C$ be the invertible function then prove that

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$$

5

(b) Prove by Mathematical Induction,

$$a + ar + ar^2 + \ldots + ar^{n-1} = \frac{a(1-r^n)}{1-r}$$

for $r \neq 1$. 5

OR

(c) Let $f : A \to B$ and $g : B \to A$ be functions such that $g \circ f = I_A$ and $f \circ g = I_B$, Then prove that f is

(d) Prove that a tree with n vertices has n – 1 edges. 5

5. (a) Define Symmetric difference of two sets, complement of Set B with respect to A and find $A \oplus B$, $B \oplus C$, if

U = {1, 2, 3, 4, 5, 6, 7, 8, 9},

A = {1, 2, 4, 6, 8}, B = {2, 4, 5, 9},

C = {x : x is positive integer and $x^2 \leq 16$}. 2½

(b) Let A = {1, 2, 3, 4, 5, 6}

Compute :

$(4, 1, 3, 5) \circ (5, 6, 3)$ 2½

(c) Define :

(i) Distributive Lattice

(ii) Complemented Lattice. 2½

(d) Define Graph. Explain the representation of graph in memory. 2½ one-to-one correspondence from A to B and g is one-to-one from B to A and each is inverse of other. 5

(d) Let A = {1, 2, 3, 4, 5, 6, 7, 8}, determine whether the permutation is even or odd,

(i) (6, 4, 2, 1, 5)

(ii) (4, 8) $\circ$ (3, 5, 2, 1) $\circ$ (2, 4, 7, 1) 5

EITHER

3. (a) Let $f : S \to T$ be a homomorphism of the semigroup $(S, *)$ onto the semigroup $(T, *)$. Let R be the relation on S defined by aRb if and only if $f(a) = f(b)$ for a and b in S. Then prove that R is a congruence relation. 5

(b) Let A = {1, 2, 3, 4, 12}. Consider a partial order divisibility on A. Draw the Hasse diagram of the poset $(A, \leq)$. 5

OR

(c) Let G be the set of all non zero real numbers and let

$$a * b = \frac{\quad}{2} \quad \forall\ a, b \in G$$

Show that $(G, *)$ is an abelian group. 5

(d) Let L be a bounded distributive lattice. Prove that if complement of $a \in L$ exists, then it is unique. 5

EITHER

4. (a) Define Euler path and Euler circuit. Prove that if a graph G has more than two vertices of odd degree, then there can be no euler path in G. 5

(b) Define :

(i) Tree

(ii) Height of tree

(iii) Complete binary tree. 5

OR

(c) Let number of edges of graph G be m, then prove that G has a Hamiltonian circuit if

$m \geq \frac{1}{2}(n^2 - 3n + 6)$ where n is the number of vertices. 5